Efficiency and sustainability
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0. This treatment is unique only in linking an explicit definition of sustainability to the concept of price elasticity of demand. For further reading, consult the sections on "price elasticity of demand," "induced demand," "rebound effect," and "Khazzoom-Brookes postulate" in an economics, ecological economics, or transportation planning textbook.

1. A sustainable system is one which persists.[1]

1.1. The sustainability of a particular system is either a prediction or a historical question. At time t we can *observe* whether a system was sustainable over the interval $[t - \tau, t]$, but we can only *predict* whether it will be sustainable over the interval $[t, t + \tau]$.[2]

1.2. One system is *more sustainable* than another if it persists, or is predicted to persist, for a longer time.

2. An economic system turns resources into goods and waste.

2.1. Resources are drawn from and waste absorbed by the ecological system of which the economic system is a part.

2.2. Suppose the economic system produces n types of goods that meet demands D_i, where $i = 1, 2, ..., n$.

2.3. For participants in the economic system, the ecological system is characterized by m types of resources. At time t there is an amount or stock $S_j(t)$ of resource j, where $j = 1, 2, ..., m$.

2.4. Absent appropriation for production, the stocks $S_j(t)$ of resources j can be modeled as

$$\frac{\partial S_j}{\partial t} = r_j S_j(t) \left[1 - \frac{S_j(t)}{K_j(t)} \right] \tag{1}$$

where r_j is the intrinsic rate of growth of resource j and $K_j(t)$ the carrying capacity for resource j at time t. Note that

$$K_j(t) = f(S_1(t), S_2(t), ..., S_{j-1}(t), S_{j+1}(t), ..., S_m(t)) \tag{2}$$

i.e., the carrying capacity for resource j at time t depends on the stock sizes of other resources. Intrinsic rates of growth and carrying capacities are determined empirically.

2.5. Suppose to produce one unit of good i the economic system uses an amount u_{ij} of resource j. The economic system's production efficiency of good i with respect to resource j is $\frac{1}{u_{ij}}$.

2.6. An amount $\sum_i u_{ij} D_i$ of resource j is required to meet demands D_i.

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[1]Costanza, R. and B. C. Patten. Defining and predicting sustainability. *Ecological Economics* **15**(3): 193-196, 1995.
[2]Ibid.

2.7. When appropriation for production occurs, resource stocks can be modeled as

$$\frac{dS_j}{dt} = \frac{\partial S_j}{\partial t} - \sum_i u_{ij} D_i(t) \tag{3}$$

2.8. Call $(\mathbf{D}, U)$ the *configuration* of the economic system at time t. If $\sum_i u_{ij} D_i > S_j(t)$ at any time t for any resource j, the configuration is unsustainable. That is, it cannot persist; literally, the desired production cannot be sustained.

2.9. Call $(\mathbf{r}, \mathcal{K})$, where $\mathcal{K}$ is the set of functions defining the carrying capacities of resources i, the configuration of the ecological system.

3. Consider two economic systems, one with production coefficients U and the other $\frac{1}{2}U$, with identical $\mathbf{D}$, $\mathbf{r}$, and $\mathcal{K}$. For empirically reasonable forms of $\mathcal{K}$, the second system is more sustainable. That is, **if demand is fixed, sustainability increases with efficiency**.

4. Instead of assuming fixed demand, suppose consumers $k = 1, 2, ..., P$ in the economic system solve the optimization problem

$$\max \mathbb{E} \int_t^\infty d_k U_k(t) dt \tag{4}$$

where their utility from consuming over the interval $[t, t + \tau]$ is, e.g.,

$$U_k(t, t + \tau) = \sum_i \int_t^\tau -(c_{ik}(t) - \hat{c}_{ik})^2 dt \tag{5}$$

i.e., they are time-discounted-utility maximizers with discount rates d_k, fixed preferences, and diminishing marginal utility from consuming any particular good.

4.1. Suppose further that demand curves consistent with these preferences interact with supply curves consistent with resource stocks $S_j(t)$ and the production coefficients u_{ij} to determine real prices and quantities of goods produced and consumed in the economic system. The form of this interaction varies in real economies, so we do not specify it here. Generally, it follows the theory of supply and demand.

4.2. Under these conditions goods will have a range of price elasticities of demand.

4.3. For a good i for which demand is not saturated and for which producers' marginal revenue is nonnegative, increases in production efficiency will lead to increases in absolute production, as more of the good can be produced for a fixed cost.

4.4. If more of the good is produced, the cost to consumers of a given amount of the good falls.

4.5. If demand for a good is elastic, this decline in cost will lead to an increase in demand and in turn to an increase in consumption.

5. Consider again two economic systems, this time with identical production coefficients U, $\mathbf{r}$,

and $\mathcal{K}$, one with demands $\mathbf{D}$ and the other $2\mathbf{D}$. For empirically reasonable forms of $\mathcal{K}$, the first system is more sustainable. That is, **sustainability falls as consumption rises**.

5.1. Therefore any change in the economic configuration which leads to increased consumption via an increase in demand leads to a decline in sustainability.

5.2. Therefore **increasing the production efficiency of a good with elastic demand leads to a decline in sustainability**.

5.3. Which goods fall in this category is an empirical question. Studies in energy economics suggest energy does. Studies in transportation suggest that demand of vehicle-miles is elastic to the price of gasoline and that demand of vehicle-miles on a particular route is elastic to the performance of links on that route. (The latter is indexed by the term "induced demand" in the transportation literature.) Computing also appears to fall in this category.

6. If long-term sustainability is the desired goal of design efforts, efficiency of computing should not be a design goal. Rather, designers should take steps to reduce demand for computing.