

# Agent-based Modelling and Participatory Simulation

## Tools for Policy Analysis After the Financial Crisis?

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### Prologue

On the morning of October 23, 2008 in the United States capital city of Washington, DC, Alan Greenspan, former Chair of the Board of Governors of the Federal Reserve, the US central bank, sat before the Committee on Oversight and Reform of the US House of Representatives, the so-called “Oversight Committee.” The official title of the hearing was “The Financial Crisis and the Role of Federal Regulators.” California Representative Henry Waxman, at that time Chair of the Oversight Committee, led the questioning. Greenspan was the first of three people called to testify. The hearing was scheduled to begin at 10:00; it appears to have started as scheduled (C-SPAN 2008).

At 10:43, Waxman asked Greenspan, “Do you have any personal responsibility for the financial crisis?”

Greenspan replied, immediately and calmly, “Let me give you a little history, Mr Chairman.”

At 10:45, Waxman interrupted Greenspan's history lesson, saying, "Dr Greenspan, I'm going to interrupt you. The question I have for you is: you had an ideology. You had a belief. This is your statement: 'I do have an ideology. My judgment is that free, competitive markets are by far the unrivaled way to organize economies. We've tried regulation; none meaningfully worked.' That was your quote. You had the authority to prevent irresponsible lending practices that led to the subprime mortgage crisis. You were advised to do so by many others. And now our whole economy is paying its price. Do you feel that your ideology pushed you to make decisions that you wish you had not made?"

Greenspan replied: "Well, remember that what an ideology is, is a conceptual framework, the way people deal with reality. Everyone has one; to exist, you need an ideology. The question is whether it is accurate or not. And what I'm saying to you is, yes, I found a flaw. I don't know how significant or permanent it is, but I've been very distressed by that fact."

Waxman: "You found a flaw... in..."

Greenspan: "A flaw in the model that I perceived in the critical functioning structure that defines how the world works, so to speak."

Waxman: "In other words, you found that your view of the world, your ideology, was not right. It was not working."

Greenspan: "Precisely. That's precisely the reason I was shocked, because I've been going for forty years or more with very considerable evidence that it was working exceptionally well."

It was 10:47.

## **1 Introduction: "All models are wrong"**

The financial crisis of 2008 took many economists and finance professionals by surprise. Indeed, the extent to which economics as a discipline essentially failed to predict the crisis was so nearly complete that the question of why this failure occurred itself became a topic of later academic discussion (e.g., Colander et al. 2009, Lawson 2009, Desai 2016, Akerlof 2018). The question was apparently perceived in some quarters to be of such great public interest that it even attracted the

attention of at least one university public relations unit (Knowledge@Wharton 2009).

Various explanations, and various remedies, have been proposed. Notably, those few economists and finance professionals who warned of an impending crisis, or even bet on it, were largely ignored; some were ridiculed. Lo (2012) for example notes how the trader John Paulson, as he first attempted to buy insurance against (in his estimation risky) collateralized debt obligations, was warned that he was making a mistake – by the very people he asked to sell him the insurance (p. 170). Nouriel Roubini, an economist who warned before the crisis of the systemic risk associated with the growth of the mortgage-backed securities market, earned the nickname “Doctor Doom” in the financial press (Lo 2012, p. 163). But why were so many economists and finance professionals so sure that Paulson, Roubini, and the few others like them were wrong – at least until they were proved right?

Part of the problem may have been attributable to perverse incentives. Many of the people who might have been able to lend their voices to the small choir of dissenters, and, perhaps, to lend it credibility in time for policymakers to take preventive action, were making money from the situation. The journalist Felix Salmon, writing in February 2009, put the matter simply: “In hindsight, ignoring those warnings looks foolhardy. But at the time, it was easy. Banks dismissed them, partly because the managers empowered to apply the brakes didn’t understand the arguments between various arms of the quant universe. Besides, they were making too much money to stop” (Salmon 2009).

### ***Failures of modelling?***

Colander et al. (2009), on the other hand, lay much of the blame not on perverse incentives but on the macroeconomic models that shaped the field’s “common sense.” In their much-discussed paper they argue that the failure to predict the crisis resulted from a systemic failure of academic economics as a discipline, and that many of the discipline’s blind spots arose from the features of its models. They observed specifically that many economic models are characterized by:

- Empirically unsupported assumptions about economic agents, including assumptions about “rationality” and “rational expectations,” and the assumption of a “representative agent” (i.e., the assumption that

economic agents can safely be assumed homogenous for modelling purposes)

- Empirically unsupported assumptions (some explicit, others implicit) about the dynamics of economic systems, including that agents are “price takers” (i.e., market power and other kinds of power are negligible in shaping market outcomes); that the effects of agents’ expectations and strategies can be assumed negligible and do not play decisive roles in market dynamics; and that markets tend to equilibrium
- “Collapse” between “micro” and “macro” levels of analysis (i.e., because of the “representative agent” approach, “the micro level *is* the macro level”)
- Limited representation of the networked structure of economic actors, including banks and other firms
- Limited endogenous representation of innovation – a crucial driver of growth and change in modern economies (see esp. Lawson 2009)
- Limited empirical testing

If this criticism of economic modelling is even partially correct, what does this mean for economic policy making?

### ***How do models influence policy making?***

To answer this question, we have to have at least a rough working theory of how economic modelling influences economic policy making. The economic sociologists Daniel Hirschman and Elizabeth Popp Berman, in a systematic literature review, approach the broader question of how economists generally influence policy. They find that direct influence is relatively limited, generally to situations in which economists are themselves “coincidentally” policy makers, or giving advice to policy makers. However, the *indirect* influence of economists on policy can be quite significant. Indirect influence includes the “professional authority” of economists generally. It also includes the prevalence of economic “styles of reasoning” among policy makers and the practical policy use of technical devices based on economic analysis. Importantly, Hirschman and Berman note, “a soft version of the economic style of reasoning is widespread among policymakers, many of

whom are exposed to it at law or policy schools” (Hirschman and Berman 2014).

In her classic study on the self-organized governance of common pool resources, the political scientist and economist Elinor Ostrom explicitly connects models, policy making, and public communication (Ostrom 1990). Some kinds of models, she notes, function as metaphors, and are invoked in public or policy discourse to argue for a particular policy – or to justify it after the fact. “When models are used as metaphors,” she writes, “an author usually points to the similarity between one or two variables in a [real world] setting and one or two variables in a model” (Ostrom 1990, pp. 7-8). For example, if a natural resource management problem can be convincingly described as a “tragedy of the commons,” policy makers and stakeholders may be inclined to believe that privatization or direct government regulation – the two classical “solutions” to “tragedies of the commons” – are the only plausible solutions the real-world policy situation (Ostrom 1990, pp. 7-8). In the language of communication research, expert models “frame” public and policy discussions of real-world situations: they determine the salient actors, environmental characteristics, and dynamics – and, in many cases, the possible solutions. Importantly, sometimes the “models” in question are very simple and generic – such as “the tragedy of the commons” – and are only “present” in a given moment of policy discussion or decision by virtue of being carried around in the heads of policy makers, experts, and other stakeholders.

One answer, then, to the question of how models influence policy is that while models may influence policy through direct use, their indirect influence may be greater. This indirect influence may occur through a variety of formal and informal channels, including formal education (including of non-economists), specialist peer-reviewed media, mass media, and even social media. Even when policy makers do not use specific models as “oracles” in real policy decisions, general models and concepts derived from modelling still influence decision making by helping decision makers “frame” decisions: when a policy maker deploys a particular model or concept in a particular situation, it can influence which actors and factors are considered relevant, what the “problem” to be addressed by policy is, and which solutions are considered possible.

If the responses to the financial crisis and other complex economic situations have been in some ways unsatisfactory, and if it seems plausible that the modelling practices of

economics as a discipline tend to contribute to rather than mitigate the problem, we can ask: How can we do economic modelling differently?

***Desiderata for future economic modelling***

If we find the analysis of Colander et al. (2009) and other critics reasonable, we can take their points, summarized very briefly above, and “turn them around” into a sort of “wish list” or list of desiderata for future economic modelling efforts. Specifically, we could strive to develop models that:

- Make only simplifying assumptions about economic agents that are consistent with empirical research in psychology, behavioral economics and finance, and other relevant fields; especially:
  - We should not assume that individual agents are “rational” in the classical sense; at most, we can assume that they are “boundedly rational” (see e.g. Simon 1957, Kahneman et al. 1982, Akerlof and Shiller 2009).
  - We should not assume that agents have “rational expectations,” i.e., that agents, even on average, understand the dynamics of the economic system they are in, or have the information required to predict its future states.
- Do not “assume away” the possible “macro” effects (self-fulfilling, self-undermining, or otherwise “performative”) of agents’ knowledge, beliefs, and strategies – effects which have been extensively documented (see e.g. MacKenzie 2007). This does not necessarily mean “building in” these effects explicitly, only ensuring that these dynamics are *possible*, and not impossible as a result of simplifying assumptions.
- Do not “assume away” market power, or other kinds of power such as bargaining power.
- Allow for heterogeneous agents; do not assume a “representative agent.”
- Ensure that micro- and macro-level dynamics do not “collapse” into each other but rather can influence

each other.

- Explicitly represent the dynamic network structure of markets and economic systems (the importance of which has been documented, especially in financial crises; see e.g., Freixas et al. 2000, Delli Gatti et al. 2005, Stiglitz and Gallegati 2011), and, as relevant, their spatial and biophysical contexts and elements.
- If possible and where relevant, represent innovation endogenously.
- Can be meaningfully tested empirically.

We are not the first writers to propose that agent-based modelling can meet at least some of these criteria. But what is agent-based modelling?

***Agent-based modelling and participatory simulation***

One useful description of agent-based modelling (ABM) is that it is an approach to simulating complex systems “from the perspective of [their] constituent units” (Bonabeau 2002).

ABM differs from differential equation modelling of complex systems in that differential equation modelling tends to focus on the behavior of and relationships between “macro” state variables, while ABM focuses on the behavior of and relationships between “agents” (see e.g. Parunak et al. 1998).

The distinction can be made clearer with an example. Consider the task of modelling the unfolding of a flu epidemic within a population. An equation-based model might describe the relationship between the numbers of susceptible people, people incubating the disease, infectious people, recovered people, and people killed by the disease. Such a model would consist of equations describing the relationships between these quantities as they change over time. Various parameters, such as for example the rate at which susceptible people become infected, or the rate with which infected people die, play decisive roles in determining the model’s prediction of how the epidemic will unfold. Some of these parameters can be estimated or derived from theoretical propositions, while others must be estimated using whatever empirical data are available.

An agent-based model of a flu epidemic, on the other hand, would not *model* the dynamics of “macro” quantities such as the number of infected persons at all. Rather, it would explicitly represent the states of, and interactions between, individual “agents.” Macro quantities and dynamics would be computed by performing calculations over the population of agents. Here, the relevant parameters are “micro” parameters, and may describe “lower level” behaviors than the macro parameters in equation based models. For example, a parameter might quantify the likelihood that a given susceptible person will leave their home and go into a space where infectious people may be present. However, if the model represents space, as many agent-based models do, whether or not that susceptible person *actually becomes infected* if they go out will depend on whether or not infected people are present in the location they go to. That is, it will depend on that person’s individual (simulated) life and environment.

In contrast, differential equation models typically “abstract from” the details of individual agents’ “lives” and interactions with other agents. The assumption behind this abstraction is that the dynamics of the system in question can be adequately modelled through relationships between aggregate variables. The approaches assumes, if implicitly, that there are no individual agents whose actions are likely to be decisive for the dynamics of the overall system, and that the actions of “unusually behaved” agents tend to “cancel each other out.” The specific structures of relationships between agents are similarly abstracted from. In agent-based modeling, individual agents’ specific actions, and the relationships between agents, are modelled explicitly, and it is possible for single agents or small groups to influence the dynamics of the entire system.

To take an economic example: an equation-based economic model concerned with inequality might model wealth inequality as a function of other macro variables such as savings, investment, productivity, output, and inflation. An agent-based model would however consider the concrete decisions of specific economic agents such as individual households, businesses, banks, and even policy makers. There would be no state variable for “wealth inequality” or even the aggregate wealth or income distributions: rather, the model would calculate these distributions by computing over the relevant quantities for each individual agent in the population.

In an agent-based model, the “macro” and “micro” levels influence each other: the values of “macro” variables are the

result of the actions and states of individual agents, and in turn constitute an “environment” that shapes individual actors’ “micro” decisions.

Participatory simulation (PS) is a kind of agent-based modelling in which some or all of the decisions of some or all of the agents are made by human participants rather than by preprogrammed computational actors. When used as a research method, participatory simulations can be understood as a kind of laboratory experiment, with similar capabilities, limitations, costs, and benefits. Indeed, while participatory simulations can theoretically be conducted online, to our knowledge, most experiments with participatory simulations thus far have been conducted in physical laboratories, with participants physically present.

***Can agent-based modelling and participatory simulation help make better policy?***

Can agent-based modelling (ABM) and participatory simulation (PS) help economists construct better models of the economy and help policy makers develop better economic policy? We believe they *can* — and we are far from the first to propose this. But we do not propose that agent-based models are better *per se* than other kinds of models.

When used as research tools (as opposed, for example, to educational or communicative tools), ABM and PS, like other research tools — for example surveys or laboratory experiments — can be used well or poorly. They have particular strengths and pose particular challenges, and they are well suited to some tasks and poorly suited to others.

ABM could be advantageous compared to other modelling approaches, or, at the very least, could be worth attempting, when:

- Previous efforts to explain or predict the dynamics of a system using inferred relationships between macro variables have met with limited success, or require many empirical “correction factors” that are difficult to reconcile with the underlying theoretical framework.
- There is some kind of “lower level” “constituent unit” in the system (i.e., the “agents”).

- There is a base of empirical knowledge that can be used to plausibly model agent behavior.

ABM may be especially advantageous when agents are heterogeneous in ways that are challenging to describe with differential equation models, especially if the actions of one or a few agents can have outsized effects, and/or when the network or spatial structure of interactions and relationships between agents may be important.

ABM poses some challenges. Because it simulates the interactions of individuals rather than of aggregate variables, ABM tends to be much more computationally intensive than comparable equation-based models. Depending on the system being modeled and the application, ABMs may require large amounts of empirical data for the setup of the initial simulation environment and for the estimation of parameters. And ABMs can be demanding to validate empirically: in the best case, both agent behavior rules and the macro patterns that emerge from agent interactions over time should be validated.

Although agent-based modelling and participatory simulation are, to our knowledge, relatively obscure in policy circles, there is a long tradition in economic applications of agent-based modelling of challenging the fundamental assumptions of mainstream economic models and policy making. Put shortly, quite a few practitioners of agent-based modelling are “fellow travellers” in the intellectual project of “rethinking capitalism.” The shorter history of participatory simulation builds on this tradition. The modest goal of this paper is merely to bring these methods and this tradition to the attention of the Regulating for Decent Work Network, and to provide a resource for researchers that highlights some possible uses of ABM and PS in pursuing the decent work agenda. With this goal in mind, the remainder of the paper proceeds simply in two long sections. The first of the two sections concerns agent-based modelling, and the second concerns participatory simulation. The section on agent-based modelling is arranged largely chronologically and can be interpreted as a sort of selective history of agent-based modelling efforts aligned with the intellectual project of “rethinking capitalism.” The second section, on participatory simulation, takes a different approach, as two of the three authors of the present paper have been directly involved in participatory simulation projects with policy implications. After a brief introduction to some of the methodological considerations involved in participatory simulation, the

sections describes one of these projects in some detail. The paper concludes with brief pointers to possible future work.

## 2 Agent-based modelling: a selective history

### ***An origin story: Thomas Schelling***

Perhaps the most well-known origin story for agent-based modelling in the social sciences begins in the 1960s with the economist Thomas Schelling. At the time – so the story goes – Schelling was interested in the phenomenon of segregation in cities. On a plane flight with nothing to read, he “began doodling with pencil and paper” (Rauch 2002). After some initial one-dimensional experiments, he expanded to two dimensions (Schelling 2006). He drew a grid with Xs and Os, and assumed that each letter would “prefer” to “live” in a square with at least two of its neighbors of the same type as itself. (Here “neighbors” include agents in squares directly above, below, to the left, and to the right, but not diagonally adjacent.) Continuing the experiments later with coins of different types on a chessboard, we found that even with seemingly mild preferences – for example, if the agents wanted only to be in a “neighborhood” of 25% similar types, meaning they didn’t want to be entirely alone – the population divided itself over time into entirely separate and homogeneous regions. In 2002, the journalist Robert Rauch wrote of the results:

“When I first looked at it, I thought I must be seeing a model of a community full of racists. I assumed, that is, that each agent wanted to live only among neighbors of its own color. I was wrong. In the simulation I’ve just described, each agent seeks only two neighbors of its own color. That is, these “people” would all be perfectly happy in an integrated neighborhood, half red, half blue. If they were real, they might well swear that they valued diversity. The realization that their individual preferences lead to a collective outcome indistinguishable from thoroughgoing racism might surprise them no less than it surprised me and, many years ago, Thomas Schelling.” (Rauch 2002)

Schelling published papers describing his models of spatial segregation in the late 1960s and early 1970s (Schelling 1969, 1971) and a book, *Micromotives and Macrobehavior*, in 1978 (Schelling 1978), which applied the same techniques to other topics; the techniques were met with interest and the 1971 paper and the book became widely cited, with many researchers building on the models (e.g., Zhang 2011) and

applying them to real-world data sets (e.g., Hatna and Benenson 2012).

***Microfoundations of cooperation: Axelrod and Hamilton***

Agent-based modeling made its second “breakthrough” in the social sciences in 1981, with the publication in *Science* of the short paper “The evolution of cooperation” by the political scientist Robert Axelrod and the evolutionary biologist W. D. Hamilton (Axelrod and Hamilton 1981). Although later widely cited in the social sciences, the paper concerned a question in evolutionary biology: how it is that cooperation among unrelated individuals, or even among individuals of different species, can emerge and persist. At the time, two explanations for cooperation dominated: kin selection and reciprocity. Kin selection cannot fully explain cooperation between unrelated individuals or individuals of different species, but theories of reciprocity had not been fully worked out. The starting point of Axelrod and Hamilton’s analysis was the “prisoner’s dilemma,” at the time already a classic thought experiment in biology and the quantitative social sciences. Their contribution was to note that in realistic scenarios, individuals might interact repeatedly, remember the outcomes of previous interactions with particular individuals, and recognize one another. They added these possibilities to the classic prisoner’s dilemma model. Then, they conducted computer-based tournaments in which agents used strategies submitted by colleagues from various disciplines. They found that “a strategy of cooperation based on reciprocity” – now famously called “tit for tat” – was “evolutionarily stable,” defeating purely selfish or exploitative strategies as long as individuals had a sufficiently large probability of continuing to interact. The legacy of this finding spans evolutionary biology and the quantitative social sciences and connects to two fundamental questions for models of economic systems: what kinds of environments are economies, and what kinds of actors populate them? Building models that answer these questions in a manner consistent with empirical findings in psychological science and behavioral economics and finance might improve models and policy outcomes.

***Institutions of cooperation: Elinor Ostrom***

In her 1990 book describing the self-organization of institutions for governance of common pool resources (Ostrom 1990), the political scientist Elinor Ostrom built on the “micro foundations” of cooperation documented by Axelrod and Hamilton to show how users of a common pool resource who

recognize that they are in a potential “prisoner’s dilemma” situation can *choose* to cooperate – and to devise arrangements to monitor and enforce that choice. That is, through cognition and communication, agents can create a “meta level” in which they collectively design both the rules under which the shared resource is to be used and the procedures that will be followed to enforce those rules.

Ostrom and her colleagues studied long-lived common pool resources in the real world, including forest, fishery, and irrigation systems, and found that many of them fit the pattern. That is, their users avoided the “tragedy of the commons” neither by privatization nor by external government control but by self-governance; rather, they devised both their own rules and the mechanisms by which they were enforced.

In later work, Ostrom identified eight institutional and biophysical features that many long-lived self-governed common pool resources shared, including: clearly defined boundaries; proportional equivalence between a user’s harvesting rights and their responsibilities to contribute to the maintenance and monitoring of the resource; the ability for many resource users to participate in rulemaking; monitoring; graduated (i.e., increasing) penalties for rule-breaking; easy access to low-cost conflict-resolution mechanisms; rights granted by higher-level authorities (e.g., government); and in some cases, multiple levels of nested self-organization (Ostrom 2005, p. 259).

How do these contributions connect to the discussion of modelling and policy making? One answer is that the concept of a long-lived, sustainably self-governed common pool resource is a new, empirically supported model that can be used as a metaphor in policy decision moments, stakeholder dialogue, and public communication.

Ostrom’s work does not only show that self-governance of common pool resources is possible. It also provides, in the form of the eight “design principles,” a sort of checklist for policy makers and resource users. Further, by developing the theoretical framework using the language of game theory and agent-based modelling, Ostrom and her colleagues made it possible to quantitatively represent and model real-world *and hypothetical* biophysical settings and institutional arrangements. The framework is sufficiently well-defined that it is possible to create computer models and laboratory experiments to explore how resource users might behave in

complex hypothetical biophysical and institutional settings. It is therefore a potentially useful tool for prospective policy analysis, stakeholder dialogue, and policy communication.

***Growing economies “from the ground up”: Epstein and Axtell***

In 1996, the social scientists Joshua Epstein and Robert Axtell published *Growing Artificial Societies*, in which they describe a series of models based on a concept they called the “Sugarscape” (Epstein and Axtell 1996). Calling their model an “artificial society” was certainly bold, but Sugarscape may have been one of the first families of models to be able to make such a claim plausibly. The basic premise is to model the dynamics of a population of agents living in a geographic space across which resources were unevenly distributed. They model one main resource – called “sugar” – that agents need to eat in order to survive. Each grid cell in the “sugarscape” is assigned a maximum amount of sugar, began the simulation with that amount, and regrows sugar up to that amount as sugar is eaten by agents. Agents, endowed at “birth” with fixed “vision” and “metabolism” (i.e., sugar requirement) attributes, search constantly in their immediate vicinity for sugar. An agent dies if it ever has zero sugar at the end of a given “time step” (i.e., if it is unable to meet its metabolic requirements).

Even with only these relatively simple rules, various dynamics “emerge.” For example, the ecological concept of “carrying capacity” – the fact that a given biophysical environment has a maximum population of organisms of a given type that it can support, appears quickly in the model. Additionally, if “seasons” are introduced in which the maximum amount of sugar in different areas changes over time, migration appears – and immigration to a sugar-rich region increases competition for sugar. And because those agents who are able to appropriate sugar in excess of their metabolic requirements are constantly accumulating sugar, there is an unequal distribution of sugar “wealth” among the agent population. And at least part of a wealthy agent’s wealth can be attributed to good genes (in the sugarscape, high vision and low metabolic requirements) and good luck (being born near areas with lots of sugar and not too many other agents).

Epstein and Axtell build a variety of phenomena on top of this basic model, including pollution, sexual reproduction with genetic mixing of heritable traits, cultural groups and interactions, conflict, trade, credit, and disease, always using only local rules and interactions. Further macro dynamics

emerge from these local rules and interactions. For example, when sexual reproduction is added to the model, endogenous population crashes become possible if the population density in a given area is too low. Genetic mixing of heritable traits also leads to the emergence – without it being programmed in – of natural selection. Under many environmental configurations, the traits of good vision and low metabolism are selected for, and come over time to dominate the agent population.

In modelling trade, Epstein and Axtell compare their models' assumptions, rules, and dynamics to those in various kinds of traditional economic models. In their initial explorations of trade on the "sugarscape," they remove sexual reproduction and assume that agents live forever and have fixed preferences. They add another kind of commodity to the original one-commodity world. They call this new commodity "spice," and stipulate that all agents also need some amount of spice in every "time step." The availability of sugar and spice is spatially variable; some areas rich in sugar may be poor in spice or vice versa. This means that some agents may, at any given moment, be rich in sugar and poor in spice, or vice versa. This creates the conditions for agents to be motivated to trade. Epstein and Axtell assume that agents only trade when it makes them better off, and model trade, in contrast to many traditional economic models but consistent with their general modelling approach, as a purely local phenomenon; they do not assume an all-knowing "Walrasian auctioneer" who intermediates all trades and allows a single equilibrium price to come into being. "Price formation is local" and exchange is completely decentralized (Epstein and Axtell 1996, p. 95).

In this context, they find that "neoclassical agents" – infinitely-lived agents with fixed preferences – are able to approach a "socially optimal" outcome from trade. That is, decentralized bilateral exchanges can approach a situation in which "no agent can be made better off through further trade" (Epstein and Axtell 1996, p. 95). When the neoclassical assumptions are relaxed, however, and replaced with more realistic assumptions – namely, agents reproduce sexually, do not live forever, and have preferences that change over time – prices do not converge to equilibrium and "the markets that emerge generally have suboptimal performance for indefinite periods of time" (ibid., p. 95).

Epstein and Axtell do not hesitate to draw implications for policy debates from these findings: "The putative case for laissez-faire economic policies is that, left to their own devices,

market processes yield equilibrium prices. Individual (decentralized) utility maximization at these prices then induces Pareto optimal allocations of goods and services. But if no price equilibrium occurs, then the efficiency of the allocations achieved becomes an open question and the theoretical case for pure market solutions is weakened” (ibid., p. 95). They note further that under even slightly realistic assumptions, it takes time for prices to converge to equilibrium, but ongoing consumption and production are always “shifting” the theoretical equilibrium price. As a result, their model economies with more realistic assumptions are *always* “far from equilibrium” (ibid., p. 116) – their dynamics are not replicated even by stastical equilibrium models in mainstream economics. As a result, Epstein and Axtell join other writers in the call for the development of a “far from equilibrium economics” (ibid., p. 137) – and in the call for such an economics to be made use of in policy making.

### ***Latter-day agent-based modelling***

A lively body of work, especially in political science, anthropology, and some branches of economics, has continued these lines of inquiry; a few illustrative examples follow. In 1999, Axtell published the first version of a working paper describing a model in which economic agents self-organize into firms according to market conditions and coordination costs (Axtell 1999; see later Axtell 2015). In 2002, Axtell, Epstein, and many other colleagues published a study describing the design and findings of an agent-based model of the collapse of the Anasazi settlements at Long House Valley in North America (Axtell et al. 2002). With a geographically accurate map and simple rules, they found that their model could reproduce fairly closely the spatial patterns of habitation revealed by the archeological record. This project, along with others, is collected in Epstein’s 2006 book *Generative Social Science* (Epstein 2006).

2006 saw a variety of publications drawing on and contributing to agent-based modelling, in particular a growing interest in models of real-world situations. The anthropologist J. Stephen Lansing, for example, published *Perfect Order* (Lansing 2006), a study of irrigation networks in Bali. Lansing and his collaborators coupled long-term ethnographic fieldwork and agent-based modelling to show how “traditional” religious structures and practices maintain a social *and* ecological balance in Bali’s agricultural societies – and how simplistic interventions designed by western development agencies,

though well-intentioned, had unexpected negative consequences when they failed to understand this intricate social, ecological, and cultural system.

The same year, Marco Janssen and Elinor Ostrom edited a special issue of the journal *Ecology and Society* on “empirically based agent based models” (Janssen and Ostrom 2006). The papers in the special issue documented models focusing on real-world settings and dynamics, including heterogeneous residential preferences and urban sprawl; agricultural policy; irrigation systems; and public goods generally.

In economics, 2006 saw the publication of the *Handbook of Computational Economics Volume 2: Agent-Based Computational Economics*, edited by Leigh Tesfatsion and Kenneth Judd (Tsfatsion and Judd, eds. 2006). The collection documents a staggering diversity of agent-based models addressing a broad range of micro- and macroeconomic phenomena, including industrial organization and firm structure, economic activity in networks, agent learning, finance, and innovation and technological change – as well as political processes and contributions making use of agent-based modelling in the *design* of markets.

Perhaps unsurprisingly, given all this, the onset of the financial crisis – and the subsequent search for explanations for the failure of economics as a discipline to predict it – saw an increase in interest in economic applications of agent-based modelling. In 2009, for example, *Science* published an opinion piece called “The economy needs agent-based modelling” (Farmer and Foley 2009). By 2014, it was possible to publish an interdisciplinary edited volume in which the gradual methodological advancement in the direction of standard approaches to “characterization and parametrization” of agent-based models was discussed at length – along with another diverse collection of models of real-world settings and phenomena including coral reef fisheries, tourism, agriculture, and energy markets (Smajgl and Barreteau, eds. 2014). And in April 2019, the OECD “New Approaches to Economic Challenges” initiative held a workshop on “New analytical tools and techniques for economic policymaking” in which agent-based modelling played a prominent role ([OECD NAEC 2019](#)). The workshop included the OECD’s chief statistician and chief economist as well as long-running contributors to agent-based modelling including Robert Axtell and J. Doyne Farmer. Indeed the NAEC initiative appears to be embracing agent-based modelling as a promising approach for economic policy support,

for example publishing a blog post explaining and advocating for it (Bookstaber 2017a; the author's most recent book is also relevant [Bookstaber 2017b]).

### **3 Participatory simulation**

Having discussed agent-based modelling at length, one way we can describe participatory simulations – or at least, those participatory simulations we are most interested in here – is that they are a class of agent-based models in which some or all of the decisions made by some or all of the simulation agents are made by human participants, rather than preprogrammed computational rules. Like agent-based models generally, they may be designed and used as research tools or as communicative or educational tools.

When used as research tools, both “non-interactive” agent-based models and participatory simulations can be considered “experiments.” The fact that participatory simulations include human participants, however, allows the researcher to ask different kinds of questions than can be asked with non-interactive agent-based models. The dynamics that emerge from the rules of a non-interactive agent-based model emerge *only* from those rules; it is exactly this emergence that agent-based experiments reveal. Human participants introduce additional complexity and uncertainty, and foreground the specifically human elements of humans' interactions in and with complex systems.

To see how, consider a hypothetical participatory version of Epstein and Axtell's Sugarscape models. A human participant could control a single agent through a game-like computer interface. At any given “time step,” the human participant would not see the entire “sugarscape” but only the part of it that “their” agent could “see,” as determined by its “genetically” predetermined vision. The participant would need to decide where to move the agent; how much of the various resources to consume given the agent's needs, available resources, and the distribution of nearby agents; whether and when to engage in conflict; whether and when to trade; when trading, how to bargain with other agents; and so on. The behavioral economics literature suggests that human participants are not likely to use optimal strategies in such situations but rather to use heuristics; generally speaking, human participants may use a much broader array of strategies than it is practical to program into a non-interactive agent

simulation. This diversity allows researchers to investigate not only the likely macro outcomes and dynamics *if* agents use particular rules or strategies but also to explore *what* kinds of rules and strategies real people might use. Thus participatory and non-interactive agent simulations can contribute to different parts of the research process.

This potential is clear from the results of computer-mediated laboratory experiments with human subjects reported by Amy Poteete, Marco Janssen, and Elinor Ostrom (Poteete et al. 2010, Ch. 6). The experiments placed participants in various common-pool resource management “dilemmas” in which existing theory predicted individuals would act selfishly, overexploiting common resources and failing to realize potential rewards from cooperation. Instead, many participants used direct communication to indicate their intention to cooperate, and many participants acted on this intention despite the possibility to defect. It would have been difficult to explicitly program the “strategies” these human participants were using into a non-interactive agent simulation – and even more difficult to justify such strategies theoretically, given that accepted theory predicted entirely different behavior. The computer mediation of the experiments allowed participants to play multiple rounds without requiring the researchers to perform complex calculations and communication tasks “manually” – that is, even though the experiments were performed in a physical laboratory, the computer mediation allowed the experiment to achieve a certain “scale.” At the same time, as in traditional experiments, concrete “implementation details” can have unexpected and significant influence over experimental outcomes, and extensive pretesting may be crucial to ensure that the experiment is measuring “the right thing.” Beyond details familiar from traditional experiments such as wording of instructions and participant incentive design, the design of participatory simulations may require knowledge and techniques from additional fields such as information visualization, cognitive ergonomics, and human-computer interaction.

Although they are still new and relatively unknown as a research tool, participatory simulations have been used in a variety of research domains, including urban logistics (Anand et al. 2016), healthcare (O’Donnell et al. 2017, Andersen and Broberg 2017), and innovation policy (Torrance and Tomlinson 2009a, 2009b, 2011). The next section describes one application in detail.

***Challenging innovation policy with a participatory simulation: The Patent Game***

Two of the present paper's coauthors – Torrance and Tomlinson – have been involved in the development and use of a participatory simulation investigating innovation policy. The simulation is called “The Patent Game” and it was used to carry out three experiments; the simulation and experiments have been documented extensively (Torrance and Tomlinson 2009a, 2009b, 2011). The experimental findings were cited in procedural documents filed before the US Supreme Court by a collection of medical associations regarding litigation relating to medical patents (American Medical Association et al. 2009). It may have been one of the first participatory simulations to become an established part of legal scholarship.<sup>1</sup>

Patents are a central tool of innovation policy, and are based on an underlying assumption that giving people or organizations monopoly exclusion rights to inventions tends to encourage scientists and other inventors to innovate. In the authors' home country of the United States, this assumption is codified both in the national constitution and in separate specific laws; similar rights exist in many other countries. The legal scholar Lawrence Lessig summarized the view that patents inspire technological innovation as follows:

“If an inventor can't get a patent, then [the inventor] will have less incentive to invent. Without a patent, [the inventor's] idea could simply be taken. If [the] idea could simply be taken, then others could benefit from [the] invention without the cost. They could, in other words, free-ride off the work of the inventor. If people could so easily free-ride, fewer would be inventors. And if fewer were inventors, then we would have less progress in 'science and useful arts.' Getting more progress is the constitutional aim of patents.” (Lessig 2001)

The purpose of the Patent Game simulation and experiments was to probe the validity of this assumption. The Patent Game is a multiplayer online business simulation in which people compete against each other to create and sell innovations, with the goal of earning money.

In designing the simulation, we operationalized the assumption we wished to test – that patent systems lead to greater social

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<sup>1</sup> Parts the following section are adapted from a previous publication (Torrance and Tomlinson 2009).

good – into a testable hypothesis: that a legal system with patents would lead to more innovation, productivity, and wealth than one without. We then devised a series of experiments that would allow us to test this hypothesis. These experiments involved groups of people interacting in a simplified version of an economy, in which various parameters could be rigorously specified, with the actions and results archived for later analysis. In order to conduct these experiments, we designed the game to simulate various legal frameworks, including patents, open source, and pure commons.

A critical aspect in scoping the interactive system to support the experiments involves specifying the key features around which the system was to be built and determining which features should be consciously omitted. In the Patent Game system, we chose to omit certain features associated with real patent systems, such as the drafting of claims, the iterative and bureaucratic process of negotiating with patent examiners, and most of the complexity of the innovation process itself. By omitting these pieces of complexity, we were able to keep the interactive experience briefer, more comprehensible, and more dynamic for the experiment participants, enhancing engagement between subjects and simulation.

The computational model for the game involves data structures for each of the following types of objects: Games, Players, Innovations, Patents, Licenses, Enforcements, and Events. For example, every Player has a name, belongs to a certain Game, owns specific Patents and Licenses, and possesses a certain amount of money. Each of the other objects has details that allows the system to operate as a simplified version of an economic system.

To enable experimental subjects to interact with the system, we created a user interface that connects with the computational models above and allows players to view various data about the simulation and enact various business strategies.

In order to study various different aspects of the planned experiments, the system archives every action taken through the interface by every player, along with a timestamp. Through this process, we can recreate the computational elements of any trial of the experiment in precise detail.

The core game play unfolds as follows: an administrator specifies the characteristics of the instance to be used in a given

run of the game, such as whether patents or open sourcing would be used; several players log in to that instance; each player concurrently makes innovations by dragging five elements ('A', 'B', 'C', 'D', and 'E') into various orders, with any ordered group of elements representing an "innovation"; these innovations are then sold to non-player consumers in exchange for some specified value. Simultaneously, if patent protection or open source is possible in a given instance, players can patent or open source different combinations, thereby affecting how other players can do business. For example, if player 1 patents combination ABC, and thereafter player 2 makes and sells combination ABCD, then player 1 can initiate litigation against player 2. Both players then specify the number of lawyers to hire, and the game determines the legal outcome probabilistically based on the number of lawyers on each side.

Each game runs for a specified amount of time, after which the winner is determined based on which individual has made the most money. In the background, the game collects information on how many unique combinations are created (a proxy for innovation), how many total combinations are made (a proxy for productivity), and how much money is earned (an unsatisfactory but simple – and, in this case, acceptable – proxy for social utility).

The interactive simulation was integrated with a broader experimental protocol. Over the course of several weeks, we invited groups of subjects to interact with the game under various conditions, analyzed the results of these interactions, and compared the rates of innovation, productivity, and social utility produced across the different systems.

Based on these experiments, we found that the pure commons system (that is, one with no intellectual property protection at all) outperformed both the pure patents system and the hybrid patent/open source system, generating more productivity and social utility, and, at a minimum, equivalent levels of innovation. That is, the experimental results call into question one of the central premises of patent law systems in place around the world – that patents promote technological progress.

Later experiments with the Patent Game explored other elements of the patent system, including the relationship of participants' understanding of the system itself on innovation (Torrance and Tomlinson 2009b) and the impact of different enforcement regimes (Torrance and Tomlinson 2011). As in the

first series of experiments investigating the effect of patenting on innovation overall, the results of these later experiments – especially the experiments on enforcement regimes – call the “received wisdom” used to justify the existing state of affairs somewhat into question. (Interested readers should see especially Torrance and Tomlinson 2011.)

We do not propose that experiments with one simulation – or various versions of one – can decisively prove or disprove propositions such as “patents stimulate innovation” or “patents promote general welfare.” It is always possible to debate the implementation of particular definitions or processes, or the external validity of particular experiments. We do believe, however, that both agent-based modelling and participatory simulation offer underexplored ways of exploring the validity of these and other policy-relevant propositions – ways that may serve researchers and policy makers especially well in the economic, political, and intellectual wake of the financial crisis.

#### **4 Conclusion: can agent-based modelling and participatory simulation help “rethink capitalism”?**

Our aim in this paper has only been to provide a resource for members of the Regulating for Decent Work network who may find the tools of agent-based modelling and participatory simulation – and the results produced by research using these tools – relevant to their existing research and policy agendas. Specifically, we believe that because ABM and PS are not constrained by the modelling conventions that came in for heavy criticism in the wake of the financial crisis, we think they may be useful in developing new models and modelling frameworks whose foundations and results are more soundly empirically supported. We believe the recent interest in ABM from international organizations such as the OECD is a promising sign, and we look forward to seeing agent-based models and participatory simulations of phenomena of direct interest of the Regulating for Decent Work Network. Indeed, we had hoped in this paper to sketch proposals for simulations of several such topics, including both relatively “specialist” topics such as regulation for digital labor markets (a much-discussed topic at the last RDW conference and at the ILO generally in the last few years) and topics of broader interest such as programs to boost green jobs and reform corporate governance. (In our home country of the United States, for example, such proposals have received increasing attention among policy makers.) We

invite other members of the RDW network interested in – or already engaged in – such an endeavor to contact us.

We should also note that agent-based modelling and participatory simulation may have significant potential in communication applications, including stakeholder dialogue, public communication, and education. We hope in future work to be able both to systematically explore the existing relevant research, for example in role-playing activities, information visualization, and digital education, that makes this potential clear, and to highlight specific potential applications. Can we imagine, for example, simulations that put students or other stakeholders in the place of policy makers in complex – and controversial – economic moments? Ultimately, “counter currents” to the intellectual propositions that brought us the financial crisis have been around for a long time; policy makers simply did not heed them. “Rethinking” can happen relatively quickly, but putting new thoughts into practice is another matter – a task not only for analysis but also for communication, including with nonspecialists. We hope in future work to explore the potential contributions these interesting methods can make to this task.

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